Universidad Simón Bolívar
Departamento de Matemáticas

Nombre: ________________________________

Carnet: ________________ Sección: 3

MA-1112 1^{er} Examen Parcial (30 %)
Intensivo 2013.

1. (8 puntos) Resuelva las siguientes integrales

   (a) (4 puntos) $\int sin(2x)\cos(2x)\sqrt{\cos(2x)+2}dx$

   Solution:

$$\begin{aligned}
\int sin(2x)\cos(2x)\sqrt{\cos(2x)+2}dx &= \frac{-1}{2}\int \cos 2x\sqrt{cos(2x)+2}(-2)\sin(2x)dx \\
&= \frac{-1}{2}\int (u-2)\sqrt{u}du = \frac{-u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3} + C \\
&= 5\frac{-(cos(2x)+2)^{\frac{5}{2}}}{5} + \frac{2(cos(2x)+2)^{\frac{3}{2}}}{3} + C
\end{aligned}$$

   (b) (4 puntos) $\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}dx$

   Solution:

$$\begin{aligned}
\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}dx &= \int_{1}^{9} \frac{(u-2)^2}{\sqrt{u}}du = \int_{1}^{9} \frac{u^2-4u+4}{u^{1/2}}du \\
&= \int_{1}^{9} (u^{3/2} - 4u^{1/2} + 4u^{-1/2})du = \frac{2}{5}u^{5/2} - 4\frac{2}{3}u^{3/2} + 4\frac{2}{1}u^{1/2}\Big|_{1}^{9} \\
&= \frac{2}{5}(9^{5/2} - 1^{5/2}) - 4\frac{2}{3}(9^{3/2} - 1^{3/2}) + 4\frac{2}{1}(9^{1/2} - 1^{1/2}) \\
&= \frac{2}{5}(242) - 4\frac{2}{3}(26) + 4\frac{2}{1}(2) \\
&= \frac{484}{5} - \frac{208}{3} + 16
\end{aligned}$$

2. (10 puntos) Calcular el área de la región acotada por la curva $y^2 = 4x + 4$ y el eje y

(a) (5 puntos) Usando el Segundo Teorema Fundamental del Cálculo

Solution:

$$\int_{-2}^{2}\left[0-\left(\frac{1}{4}x^2-1\right)\right]dx = 2\int_0^2\left(1-\frac{1}{4}x^2\right)dx = 2\left(x-\frac{x^3}{12}\right)\Big|_0^2 = 2\left(2-\frac{8}{12}\right) = \frac{8}{3}$$

(b) (5 puntos) Usando sumas de Riemann

Solution: Sean $x_i = i\frac{2}{n}$ para $i = 1, 2, \ldots, n$ y $\Delta x = \frac{2}{n}$

$$\begin{aligned}
\lim_{n\to\infty} -2\sum_{i=1}^{n} f(x_i)\Delta x &= -2\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{1}{4}x_i^2-1\right)\frac{2}{n} \\
&= -2\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{1}{4}\left(i\frac{2}{n}\right)^2-1\right]\frac{2}{n} \\
&= -2\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{i^2 8}{4n^3}-\sum_{i=1}^{n}\frac{2}{n}\right) \\
&= -2\lim_{n\to\infty}\left(\frac{8}{4n^3}\frac{n(n+1)(2n+1)}{6}-2\right) \\
&= -2\left(\frac{2}{3}-2\right) = \frac{8}{3}
\end{aligned}$$

3. (7 puntos) Demuestre la desigualdad

$$\frac{\sqrt{2}}{6} \leq \int_0^1 \frac{x^2}{\sqrt{1+x}} \leq \frac{1}{3}$$

Sugerencia: acote $\frac{x^2}{\sqrt{1+x}}$ inferiormente por $\frac{x^2}{\sqrt{2}}$

Solution: La función es continua en $[0, 1]$. Por lo cual

$$\frac{x^2}{\sqrt{2}} \leq \frac{x^2}{\sqrt{1+x}} \leq x^2$$

Usando el segundo Teorema Fundamental del Calculo tenemos

$$\frac{1}{2}\int_0^1 x^2 dx \leq \int_0^1 \frac{x^2}{\sqrt{1+x}}dx \leq \int_0^1 x^2 dx$$

$$\frac{1}{3\sqrt{2}} \leq \int_0^1 \frac{x^2}{\sqrt{1+x}}dx \leq \frac{1}{3}$$

$$\boxed{\frac{\sqrt{2}}{6} \leq \int_0^1 \frac{x^2}{\sqrt{1+x}}dx \leq \frac{1}{3}}$$